CHAPTER 6 REFLECTOR ANTENNAS

Helmut E. Schrank Gary E. Evans Daniel Davis

Electronic Systems Group Westinghouse Electric Corporation

6.1 INTRODUCTION

The basic role of the radar antenna is to provide a Role of the Antenna. transducer between the free-space propagation and the guided-wave propagation of electromagnetic waves. The specific function of the antenna during transmission is to concentrate the radiated energy into a shaped directive beam which illuminates the targets in a desired direction. During reception the antenna collects the energy contained in the reflected target echo signals and delivers it to the receiver. Thus the radar antenna is used to fulfill reciprocal but related roles during its transmit and receive modes. In both of these modes or roles, its primary purpose is to accurately determine the angular direction of the target. For this purpose, a highly directive (narrow) beamwidth is needed, not only to achieve angular accuracy but also to resolve targets close to one another. This important feature of a radar antenna is expressed quantitatively in terms not only of the beamwidth but also of transmit gain and effective receiving aperture. These latter two parameters are proportional to one another and are directly related to the detection range and angular accuracy. Many radars are designed to operate at microwave frequencies, where narrow beamwidths can be achieved with antennas of moderate physical size.

The above functional description of radar antennas implies that a single antenna is used for both transmitting and receiving. Although this holds true for most radar systems, there are exceptions: some monostatic radars use separate antennas for the two functions; and, of course, bistatic radars must, by definition, have separate transmit and receive antennas. In this chapter, emphasis will be on the more commonly used single antenna and, in particular, on the widely used reflector antennas. Phased array antennas are covered in Chap. 7.

Beam Scanning and Target Tracking. Because radar antennas typically have directive beams, coverage of wide angular regions requires that the narrow beam be scanned rapidly and repeatedly over that region to assure detection of targets wherever they may appear. This describes the *search* or *surveillance function* of a radar. Some radar systems are designed to follow a target once it has been detected, and this *tracking function* requires a specially designed antenna different from a surveillance radar antenna. In some radar systems, particularly airborne radars, the antenna is designed to perform both search and track functions.

Height Finding. Most surveillance radars are two-dimensional (2D), measuring only range and azimuth coordinates of targets. In early radar systems, separate height finding antennas with mechanical rocking motion in elevation were used to determine the third coordinate, namely, elevation angle, from which the height of an airborne target could be computed. More recent designs of three-dimensional (3D) radars use a single antenna to measure all three coordinates: for example, an antenna forming a number of stacked beams in elevation in the receive mode and a single broad-coverage elevation beam in the transmit mode. The beams are all equally narrow in azimuth, but the vertically stacked receive beams allow measuring echo amplitudes in two adjacent overlapping beams to determine the elevation angle of the target.

Classification of Antennas. Radar antennas can be classified into two broad categories, *optical antennas* and *array antennas*. The optical category, as the name implies, comprises antennas based on optical principles and includes two subgroups, namely, *reflector antennas* and *lens antennas*. Reflector antennas are still widely used for radar, whereas lens antennas, although still used in some communication and electronic warfare (EW) applications, are no longer used in modern radar systems. For that reason and to reduce the length of this chapter, lens antennas will not be discussed in detail in this edition of the handbook. However, references on lens antennas from the first edition will be kept in the list at the end of the chapter.

6.2 BASIC PRINCIPLES AND PARAMETERS

This section briefly reviews basic antenna principles with emphasis on definitions of terms useful to a radar system designer. In order to select the best type of antenna for a radar system, the system designer should have a clear understanding of the basic performance features of the wide variety of antenna types from which he or she must choose.¹ This includes the choice between reflector antennas, covered in this chapter, and phased arrays, covered in Chap. 7. Another alternative is a reflector fed by a phased array.

Although the emphasis in this chapter is on reflectors, many of the basic principles discussed in this section apply to all antennas. The three basic parameters that must be considered for any antenna are:

- Gain (and effective aperture)
- Radiation pattern (including beamwidth and sidelobes)
- Impedance (voltage-standing-wave ratio, or VSWR)

Other basic considerations are *reciprocity* and *polarization*, which will be briefly discussed in this section.

Reciprocity. Most radar systems employ a single antenna for both transmitting and receiving, and most such antennas are reciprocal devices, which means that their performance parameters (gain, pattern, impedance) are identical for the two functions. This reciprocity principle² allows the antenna to be considered either as a transmitting or as a receiving device, whichever is

more convenient for the particular discussion. It also allows the antenna to be tested in either role (Sec. 6.10).

Examples of *nonreciprocal* radar antennas are phased arrays using nonreciprocal ferrite components, active arrays with amplifiers in the transmit/receive (T/R) modules, and height finding antennas for 3D (range, azimuth, and elevation) radars. The last-named, typified by the AN/TPS-43³ radar, uses several overlapping beams stacked in elevation for receiving with a broad elevation beam for transmitting. All beams are equally narrow in the azimuth direction. These nonreciprocal antennas must be tested separately for their transmitting and receiving properties.

Gain, Directivity, and Effective Aperture. The ability of an antenna to concentrate energy in a narrow angular region (a directive beam) is described in terms of antenna gain. Two different but related definitions of antenna gain are *directive gain* and *power gain*. The former is usually called *directivity*, while the latter is often called *gain*. It is important that the distinction between the two be clearly understood.

Directivity (directive gain) is defined as the maximum radiation intensity (watts per steradian) relative to the average intensity, that is,

$$G_D = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}} = \frac{\text{maximum power per steradian}}{\text{total power radiated/}4\pi}$$
(6.1)

This can also be expressed in terms of the maximum radiated-power density (watts per square meter) at a far-field distance R relative to the average density at that same distance:

$$G_D = \frac{\text{maximum power density}}{\text{total power radiated}/4\pi R^2} = \frac{p_{\text{max}}}{P_t/4\pi R^2}$$
(6.2)

Thus the definition of directivity is simply how much stronger the actual maximum power density is than it would be if the radiated power were distributed isotropically. Note that this definition does not involve any dissipative losses in the antenna but only the concentration of *radiated power*.

Gain (power gain) does involve antenna losses and is defined in terms of power accepted by the antenna at its input port P_0 rather than radiated power P Thus gain is given by

$$G = \frac{\text{maximum power density}}{\text{total power accepted/4}\pi R^2} = \frac{p_{\text{max}}}{P_0/4\pi R^2}$$
(6.3)

For realistic (nonideal) antennas, the power radiated P_t is equal to the power accepted P_0 times the *radiation efficiency* factor η of the antenna:

$$P_t = \eta P_0 \tag{6.4}$$

As an example, if a typical antenna has 1.0 dB dissipative losses, $\eta = 0.79$, and it will radiate 79 percent of its input power. The rest, $(1 - \eta)$ or 21 percent, is converted into heat. For reflector antennas, most losses occur in the transmission line leading to the feed and can be made less than 1 dB.

By comparing Eqs. (6.2) and (6.3) with (6.4), the relation between gain and directivity is simply

$$G = \eta G_D \tag{6.5}$$

Thus antenna gain is always less than directivity except for ideal lossless antennas, in which case $\eta = 1.0$ and $G = G_D$.

Approximate Directivity—Beamwidth Relations. An approximate but useful relationship between directivity and antenna beamwidths (see Sec. 2.3) is

$$G_D \simeq \frac{40,000}{B_{\rm az}B_{\rm el}} \tag{6.6}$$

where B_{az} and B_{el} are the principal-plane azimuth and elevation half-power beamwidths (in degrees), respectively. This relationship is equivalent to a 1° by 1° pencil beam having a directivity of 46 dB. From this basic combination, the approximate directivities of other antennas can be quickly derived: for example, a 1° by 2° beam corresponds to a directivity of 43 dB because doubling one beamwidth corresponds to a 3 dB reduction in directivity. Similarly, a 2° by 2° beam has 40 dB, a 1° by 10° beam has 36 dB directivity, and so forth. Each beamwidth change is translated into decibels, and the directivity is adjusted accordingly. This relation does not apply to shaped (e.g., cosecant-squared) beams.

Effective Aperture. The aperture of an antenna is its physical area projected on a plane perpendicular to the main-beam direction. The concept of effective aperture is useful when considering the antenna in its receiving mode. For an ideal (lossless), uniformly illuminated aperture of area A operating at a wavelength λ , the directive gain is given by

$$G_D = 4\pi A/\lambda^2 \tag{6.7}$$

This represents the *maximum available gain* from an aperture A and implies a perfectly flat phase distribution as well as uniform amplitude.

Typical antennas are not uniformly illuminated but have a tapered illumination (maximum in the center of the aperture and less toward the edges) in order to reduce the sidelobes of the pattern. In this case, the directive gain is less than that given by Eq. (6.7):

$$G_D = 4\pi A_e / \lambda^2 \tag{6.8}$$

where A_e is the effective aperture or *capture area* of the antenna, less than the physical aperture A by a factor ρ_a usually called the *aperture efficiency*.

$$A_e = \rho_a A \tag{6.9}$$

This aperture efficiency would better be called *aperture effectiveness* because it does not involve RF power turned into heat; i.e., it is not a dissipative effect but only a measure of how effectively a given aperture is utilized. An antenna with an aperture efficiency of, say, 50 percent ($\rho_a = 0.5$) has a gain 3 dB below the uniformly illuminated aperture gain but does not dissipate half the power involved. The effective aperture represents a smaller, uniformly illuminated aperture having the same gain as that of the actual, nonuniformly illuminated aperture. It is the area which, when multiplied by the incident power density P_i , gives the power received by the antenna:

$$P_r = P_i A_e \tag{6.10}$$

Radiation Patterns. The distribution of electromagnetic energy in threedimensional angular space, when plotted on a relative (normalized) basis, is called the *antenna radiation pattern*. This distribution can be plotted in various ways, e.g., polar or rectangular, voltage intensity or power density, or power per unit solid angle (radiation intensity). Figure 6.1 shows a typical radiation pattern for a circular-aperture antenna plotted isometrically in terms of the logarithmic power density (vertical dimension in decibels) versus the azimuth and elevation angles in rectangular coordinates. The *main lobe* (or *main beam*) of the pattern is a *pencil beam* (circular cross section) surrounded by *minor lobes*, commonly referred to as *sidelobes*. The angular scales have their origins at the peak of the main lobe, which is generally the *electrical reference axis* of the antenna.

This axis may or may not coincide with the *mechanical axis* of the antenna, i.e., the axis of symmetry, sometimes called the *boresight axis*. If the two do not coincide, which usually happens unintentionally, the angular difference is referred to as a *boresight error* and must be accounted for in the measurement of target directions.

Figure 6.1*a* shows the three-dimensional nature of all antenna patterns, which require extensive data to plot in this form. This same data can also be plotted in the form of constant-power-level contours, as shown in Fig. 6.1*c*. These contours are the intersections of a series of horizontal planes through the 3D pattern at various levels and can be quite useful in revealing the distribution of power in angular space.

More frequently, however, 2D plots are sufficient and more convenient to measure and plot. For example, if the intersection of the 3D pattern of Fig. 6.1a with a vertical plane through the peak of the beam and the zero azimuth angle is taken, a 2D slice or "cut" of the pattern results, as shown in Fig. 6.1b. This is called the *principal-plane elevation pattern*. A similar cut by a vertical plane or



FIG. 6.1 Typical pencil-beam pattern. (a) Three-dimensional cartesian plot of complete pattern.



FIG. 6.1 (Continued) (b) Principal-plane elevation pattern. (c) Contours of constant intensity (isophotes). (Courtesy of D. D. Howard, Naval Research Laboratory.)

thogonal to the first one, i.e., containing the peak and the 0° elevation angle, results in the so-called *azimuth pattern*, also a principal-plane cut because it contains the peak of the beam as well as one of the angular coordinate axes.

These principal planes are sometimes called *cardinal planes*. All other vertical planes through the beam peak are called *intercardinal planes*. Sometimes pattern cuts in the $\pm 45^{\circ}$ intercardinal planes are measured and plotted, but most often it is sufficient to plot only the azimuth and elevation patterns to describe the pattern performance of an antenna. In other words, it is sufficient (and much less costly) to *sample* the 3D pattern with two planar cuts containing the beam axis.

The terms azimuth and elevation imply earth-based reference coordinates, which may not always be applicable, particularly to an airborne or space-based (satellite) system. A more generic pair of principal planes for antennas in general are the so-called E and H plane of a linearly polarized antenna. Here the E-plane pattern is a principal plane containing the direction of the electric (E-field) vector of the radiation from the antenna, and the H plane is orthogonal to it, therefore containing the magnetic (H-field) vector direction. These two principal planes can be independent of earth-oriented directions such as azimuth and elevation and are widely used.

It should be noted that sampling 3D antenna patterns is not limited to planar cuts as described above. Sometimes it is meaningful and convenient (from a measuring-technique viewpoint) to take *conical cuts*, i.e., the intersections of the 3D pattern with cones of various angular widths centered on the electrical (or mechanical) axis of the antenna.

The typical 2D pattern shown in Fig. 6.1*b* is plotted in rectangular coordinates, with the vertical axis in decibels. This is by far the most widely used form of plotting patterns because it provides a wide dynamic range of pattern levels with good visibility of the pattern details. However, other forms of plotting-pattern data are also used, as illustrated in Fig. 6.2. This shows four forms of plotting the same sin x/x pattern, including (*a*) a polar plot of relative voltage (intensity), (*b*) a rectangular plot of voltage, (*c*) a rectangular plot of relative power (density), and (*d*) a rectangular plot of logarithmic power (in decibels). The linear voltage and power scales in *a*, *b*, and *c* leave much to be desired in showing lower-level pattern details, while *d* provides good "visibility" of the entire pattern. Of course, polar patterns can also be plotted by using decibels in the radial dimension, but lower-level details are compressed near the center of the pattern chart and visibility is very poor. Figure 6.2 shows the reason why rectangular-decibel pattern plots are most often preferred.

Beamwidth. One of the main features of an antenna pattern is the beamwidth of the main lobe, i.e., its angular extent. However, since the main beam is a continuous function, its width varies from the peak to the nulls (or minima). The most frequently expressed width is the half-power beamwidth (HPBW), which occurs at the 0.707-relative-voltage level in Fig. 6.2a and b, at the 0.5-relative-power level in c, and at the 3 dB level in d. Sometimes other beamwidths are specified or measured, such as the one-tenth power (10 dB) beamwidth, or the width between nulls, but unless otherwise stated the simple term beamwidth implies the half-power (3 dB) width. This half-power width is also usually a measure of the resolution of an antenna, so that two identical targets at the same range are said to be resolved in angle if separated by the half-power beamwidth.

The beamwidth of an antenna depends on the size of the antenna aperture as well as on the amplitude and phase distribution across the aperture. For a given distribution, the beamwidth (in a particular planar cut) is inversely proportional to the size of the aperture (in that plane) expressed in wavelengths. That is, the half-power beamwidth is given by

$$HPBW = K/(D/\lambda) = K\lambda/D$$
(6.11)

where D is the aperture dimension, λ is the free-space wavelength, and K is a proportionality constant known as the *beamwidth factor*. Each amplitude distribution (assuming a linear phase distribution) has a corresponding beamwidth factor, expressed either in radians or in degrees.



FIG. 6.2 Various representations of the same sin x/x pattern.

Sidelobes. The lobe structure of the antenna radiation pattern outside the major-lobe (main-beam) region usually consists of a large number of *minor lobes*, of which those adjacent to the main beam are *sidelobes*. However, it is common usage to refer to all minor lobes as sidelobes, in which case the adjacent lobes are called *first sidelobes*. The minor lobe approximately 180° from the main lobe is called the *backlobe*.

Sidelobes can be a source of problems for a radar system. In the transmit mode they represent wasted radiated power illuminating directions other than the desired main-beam direction, and in the receive mode they permit energy from undesired directions to enter the system. For example, a radar for detecting lowflying aircraft targets can receive strong ground echoes (*clutter*) through the sidelobes which mask the weaker echoes coming from low radar cross-section targets through the main beam. Also, unintentional interfering signals from friendly sources (electromagnetic interference, or EMI) and/or intentional interference from unfriendly sources (jammers) can enter through the minor lobes. It is therefore often (but not always) desirable to design radar antennas with sidelobes as low as possible (consistent with other considerations) to minimize such problems. (NOTE: There are systems in which the lowest possible sidelobes are not desirable; for example, to minimize main-beam clutter or jamming, it may be better to tolerate higher sidelobes in order to achieve a narrower main-beam null width).

To achieve low sidelobes, antennas must be designed with special tapered amplitude distributions across their apertures. For a required antenna gain this means that a larger antenna aperture is needed. Conversely, for a given size of antenna, lower sidelobes mean less gain and a correspondingly broader beamwidth. The optimum compromise (tradeoff) between sidelobes, gain, and beamwidth is an important consideration for choosing or designing radar antennas. Figure 7.23 of Chap. 7 shows these tradeoff relations for the optimum Taylor amplitude distribution^{4.5} being widely used for sidelobe suppression in radar antennas. One set of curves is for rectangular (linear) apertures; the other, for circular Taylor distributions.

The sidelobe levels of an antenna pattern can be specified or described in several ways. The most common expression is the *relative sidelobe level*, defined as the peak level of the highest sidelobe relative to the peak level of the main beam. For example, a "-30 dB sidelobe level" means that the peak of the highest sidelobe has an intensity (radiated power density) one one-thousandth (10⁻³ or -30 dB) that of the peak of the main beam. Sidelobe levels can also be quantified in terms of their *absolute level* relative to isotropic. In the above example, if the antenna gain were 35 dB, the absolute level of the -30 dB relative sidelobe is +5dBi, i.e., 5 dB above isotropic. For some radar systems, the peak levels of individual sidelobes are not as important as the average level of all the sidelobes. This is particularly true for airborne "down-look" radars like the Airborne Warning and Control System, or AWACS (E-3A), which require very low (ultralow) average sidelobe levels in order to suppress ground clutter. The average level is a power average (sometimes referred to as the rms level) formed by integrating the power in all minor lobes outside the main lobe and expressing it in decibels relative to isotropic (dBi). For example, if 90 percent of the power radiated is in the main beam, 10 percent (or 0.1) is in all the sidelobes: this corresponds to an average sidelobe level of -10 dBi. If the main beam contains 99 percent of the radiated power, the average sidelobe level is 0.01, or -20 dBi, etc. Ultralow average sidelobe levels, defined as better than -20 dBi, have been achieved with careful design and manufacturing processes.

One other way to describe sidelobe levels (not often used but sometimes meaningful) is by the *median level*; this is the level such that half of the angular space has sidelobe levels above it and the other half has them below that level.

Polarization. The direction of polarization of an antenna is defined as the direction of the electric-field (*E*-field) vector. Many existing radar antennas are *linearly polarized*, usually either *vertically* or *horizontally*; although these designations imply an earth reference, they are quite common even for airborne or satellite antennas.

Some radars use *circular polarization* in order to detect aircraftlike targets in rain. In that case, the direction of the E field varies with time at any fixed observation point, tracing a circular locus once per RF cycle in a fixed plane normal to the direction of propagation. Two senses of circular polarization (CP) are possible, right-hand (RHCP) and left-hand (LHCP). For RHCP, the electric vector appears to rotate in a clockwise direction when viewed as a wave receding from the observation point. LHCP corresponds to counterclockwise rotation. These definitions of RHCP and LHCP can be illustrated with hands, by pointing the thumb in the direction of propagation and curling the fingers in the apparent direction of E-vector rotation. By reciprocity, an antenna designed to radiate a particular polarization will also receive the same polarization.

The most general polarization is *elliptical polarization* (EP), which can be thought of as imperfect CP such that the *E* field traces an ellipse instead of a circle. A clear discussion of polarizations can be found in Kraus.^{6,7}

Another increasingly important consideration for radar antennas is not only what polarization they radiate (and receive) but how pure their polarization is. For example, a well-designed vertical-polarization antenna may also radiate small amounts of the orthogonal horizontal polarization in some directions (usually off the main-beam axis). Similarly, an antenna designed for RHCP many also radiate some LHCP, which is mathematically orthogonal to RHCP. The desired polarization is referred to as the main polarization (COPOL), while the undesired orthogonal polarization is called cross polarization (CROSSPOL). Polarization purity is important in the sidelobe regions as well as in the main-beam region. Some antennas with low COPOL sidelobes may, if not properly designed, have higher CROSSPOL sidelobes, which could cause clutter or jamming problems. A welldesigned antenna will have CROSSPOL components at least 20 dB below the COPOL in the main-beam region, and 5 to 10 dB below in the sidelobe regions. Reflecting surfaces near an antenna, such as aircraft wings or a ship superstructure, can affect the polarization purity of the antenna, and their effects should be checked.

6.3 TYPES OF ANTENNAS

Reflector antennas are built in a wide variety of shapes with a corresponding variety of feed systems to illuminate the surface, each suited to its particular application. Figure 6.3 illustrates the most common of these, which are described in some detail in the following subsections. The paraboloid in Fig. 6.3a collimates radiation from a feed at the focus into a pencil beam, providing high gain and minimum beamwidth. The parabolic cylinder in Fig. 6.3b performs this



FIG. 6.3 Common reflector antenna types. (a) Paraboloid. (b) Parabolic cylinder. (c) Shaped. (d) Stacked beam. (e) Monopulse. (f) Cassegrain. (g) Lens.

collimation in one plane but allows the use of a linear array in the other plane, thereby allowing flexibility in the shaping or steering of the beam in that plane. An alternative method of shaping the beam in one plane is shown in Fig. 6.3c, in which the surface itself is no longer a paraboloid. This is a simpler construction, but since only the phase of the wave across the aperture is changed, there is less control over the beam shape than in the parabolic cylinder, whose linear array may be adjusted in amplitude as well.

Very often the radar designer needs multiple beams to provide coverage or to determine angle. Figure 6.3d shows that multiple feed locations produce a set of secondary beams at distinct angles. The two limitations on adding feeds are that they become defocused as they necessarily move away from the focal point and that they increasingly block the aperture. An especially common multiple-beam design is the monopulse antenna of Fig. 6.3e, used for angle determination on a single pulse, as the name implies. In this instance the second beam is normally a difference beam with its null at the peak of the first beam.

Multiple-reflector systems, typified by the Cassegrain antenna of Fig. 6.3f, offer one more degree of flexibility by shaping the primary beam and allowing the feed system to be conveniently located behind the main reflector. The symmetrical arrangement shown has significant blockage, but offset arrangements can readily be envisioned to accomplish more sophisticated goals.

Lenses (Fig. 6.3g) are not as popular as they once were, largely because phased arrays are providing many functions that lenses once fulfilled. Primarily they avoid blockage, which can become prohibitive in reflectors with extensive feed systems. A very wide assortment of lens types has been studied.⁸⁻¹³

In modern antenna design, combinations and variations of these basic types are widespread, with the goal of minimizing loss and sidelobes while providing the specified beam shapes and locations.



FIG. 6.4 Geometrical representation of a paraboloidal reflector. (a) Geometry. (b) Operation.

Paraboloidal Reflector Antennas. The theory and design of paraboloidal reflector antennas are extensively discussed in the literature.^{2-4,14,15} The basic geometry is that of Fig. 6.4*a*, which assumes a parabolic conducting reflector surface of focal length f with a feed at the focus F. It can be shown from geometrical optics considerations that a spherical wave emerging from F and incident on the reflector is transformed after reflection into a plane wave traveling in the positive z direction (Fig. 6.4*b*).

The two coordinate systems that are useful in analysis are shown in Fig. 6.4*a*. In rectangular coordinates (x, y, z), the equation of the paraboloidal surface with a vertex at the origin (0, 0, 0) is

$$z = (x^2 + y^2)/4f$$
(6.12)

In spherical coordinates $(\rho,\,\psi,\,\xi)$ with the feed at the origin, the equation of the surface becomes

$$\rho = f \sec^2 \frac{\Psi}{2} \tag{6.13}$$

This coordinate system is useful for designing the pattern of the feed. For example, the angle subtended by the edge of the reflector at the feed can be found from

$$\tan\frac{\Psi_0}{2} = D/4f \tag{6.14}$$

The aperture angle $2\psi_0$ is plotted as a function of f/D in Fig. 6.5. Reflectors with the longer focal lengths, which are flattest and introduce the least distortion of polarization and of off-axis beams, require the narrowest primary beams and therefore the largest feeds. For example, the size of a horn to feed a reflector of f/D = 1.0 is approximately 4 times that of a feed for a reflector of f/D = 0.25. Most reflectors are chosen to have a focal length between 0.25 and 0.5 times the diameter.



FIG. 6.5 Subtended angle of the edge of a paraboloidal reflector.

When a feed is designed to illuminate a reflector with a particular taper, the distance ρ to the surface must be accounted for, since the power density in the spherical wave falls off as $1/\rho^2$. Thus the level at the edge of the reflector will be lower than at the center by the product of the feed pattern and this "space taper." The latter is given in decibels as

Space taper (dB) =
$$20 \log \frac{(4f/D)^2}{1 + (4f/D)^2}$$
 (6.15)

Equation (6.15) is graphed in Fig. 6.6, showing a significant contribution at the smaller focal lengths. In low-sidelobe applications this amplitude variation may be used in conjunction with the feed pattern to achieve a specific shaping to the skirts of the distribution across the aperture.

Although this reflector is commonly illustrated as round with a central feed point, a variety of reflector outlines are in use, as shown in Fig. 6.7. Often, the

RADAR HANDBOOK



FIG. 6.6 Edge taper due to the spreading of the spherical wave from the feed ("space loss").



FIG. 6.7 Paraboloidal reflector outlines. (a) Round. (b) Oblong. (c) Offset feed. (d) Mitered corner. (e) Square corner. (f) Stepped corner.

azimuth and elevation beamwidth requirements are quite different, requiring the "orange-peel," or oblong, type of reflector of Fig. 6.7b.

As sidelobe levels are reduced and feed blockage becomes intolerable, offset feeds (Fig. 6.7c) become necessary. The feed is still at the focal point of the portion of the reflector used even though the focal axis no longer intersects the reflector. Feeds for an offset paraboloid must be aimed beyond the center of the reflector area to account for the larger space taper on the side of the dish away from the feed. The result is an unsymmetrical illumination.

The corners of most paraboloidal reflectors are rounded or mitered as in Fig. 6.7d to minimize the area and especially to minimize the torque required to turn

the antenna. The deleted areas have low illumination and therefore least contribution to the gain. However, circular and elliptical outlines produce sidelobes at all angles from the principal planes. If low sidelobes are specified away from the principal planes, it may be necessary to maintain square corners, as shown in Fig. 6.7e.

Parabolic reflectors still serve as a basis for many radar antennas in use today, since they provide the maximum available gain and minimum beamwidths with the simplest and smallest feeds.

Parabolic-Cylinder Antenna.^{2,16,17} It is quite common that either the elevation or the azimuth beam must be steerable or shaped while the other is not. A parabolic cylindrical reflector fed by a line source can accomplish this flexibility at a modest cost. The line source feed may assume many different forms ranging from a parallel-plate lens to a slotted waveguide to a phased array using standard designs.²⁻⁴

The parabolic cylinder has application even where both patterns are fixed in shape. The AN/TPS-63 (Fig. 6.8) is one such example in which elevation beam



FIG. 6.8 AN/TPS-63 parabolic-cylinder antenna. (Courtesy Westinghouse Electric Corporation.)

shaping must incorporate a steep skirt at the horizon to allow operation at low elevation angles without degradation from ground reflection. A vertical array can produce much sharper skirts than a shaped dish of equal height can, since a shaped dish uses part of its height for high-angle coverage. The array can superimpose high and low beams on a common aperture, thereby benefiting from the full height for each.

The basic parabolic cylinder is shown in Fig. 6.9, in which the reflector surface has the contour

$$z = y^2/4f$$
 (6.16)

The feed is on the focal line F-F', and a point on the reflector surface is located with respect to the feed center at x and $\rho = f \sec^2 \psi/2$. Many of the guidelines for paraboloids except space taper can be carried over to parabolic cylinders. Since the feed energy diverges on a cylinder instead of a sphere, the power density falls off as ρ rather than ρ^2 . Therefore, the space taper of Eq. (6.15) is halved in decibels.



FIG. 6.9 Parabolic cylinder. (a) Geometry. (b) Extension.

The height or length of the parabolic cylinder must account for the finite beamwidth, shaping, and steering of the linear feed array. As Fig. 6.9 indicates, at angle θ from broadside the primary beam intercepts the reflector at $f \tan \theta$ past the end of the vertex. Since the peak of the primary beam from a steered line source lies on a cone, the corresponding intercepts on the right and left corners of the top of reflector are farther out at $f \sec^2 \psi_0/2 \tan \theta$. For this reason, the corners of a parabolic cylinder are seldom rounded in practice.

Parabolic cylinders suffer from large blockage if they are symmetrical, and they are therefore often built offset. Properly designed, however, a cylinder fed by an offset multiple-element line source can have excellent performance¹⁸ (Fig. 6.10). A variation on this design has the axis of the reflector horizontal, fed with



FIG. 6.10 Pillbox structure used to test a low-sidelobe parabolic cylinder and its measured pattern. (Courtesy Ronald Fante, Rome Air Development Center.)

a linear array for low-sidelobe azimuth patterns and shaped in height for elevation coverage. It is an economical alternative to a full two-dimensional array.

Shaped Reflectors. Fan beams with a specified shape are required for a variety of reasons. The most common requirement is that the elevation beam

provide coverage to a constant altitude. If secondary effects are ignored and if the transmit and receive beams are identical, this can be obtained with a power radiation pattern proportional to $\csc^2\theta$, where θ is the elevation angle.^{2,19} In practice, this well-known cosecant-squared pattern has been supplanted by similar but more specific shapes that fit the earth's curvature and account for sensitivity time control (STC).

The simplest way to shape the beam is to shape the reflector, as Fig. 6.11 illustrates. Each portion of the reflector is aimed in a different direction and, to the extent that geometric optics applies, the amplitude at that angle is the integrated sum of the power density from the feed across that portion. Silver² describes the procedure to determine the contour for a cosecant-squared beam graphically. However, with modern computers arbitrary beam shapes can be approximated accurately by direct integration of the reflected primary pattern. In so doing, the designer can account for the approximations to whatever accuracy he or she chooses. In particular, the azimuth taper of the primary beam can be included,



FIG. 6.11 Reflector shaping.



FIG. 6.12 Three-dimensional shaped-reflector design.

and the section of the reflector aimed at elevation θ can be focused in azimuth and have a proper outline when viewed from elevation θ (Fig. 6.12). Without these precautions off-axis sidelobes can be generated by banana-shaped sections.

Most shaped reflectors take advantage of the shaping to place the feed outside the secondary beam. Figure 6.13 shows how blockage can be virtually eliminated even though the feed appears to be opposite the reflector.

The ASR-9 (Fig. 6.14) typifies shaped reflector antennas designed by these procedures. The elevation shaping, azimuth beam skirts, and sidelobes are closely controlled by the use of the computer-aided design process.

A limitation of shaped reflectors is that a large fraction of the aperture is not used in forming the main beam. If the feed pattern is symmetrical and half of the power is directed to wide angles, it follows that the main beam



FIG. 6.13 Elimination of blockage.

will use half of the aperture and have double the beamwidth. This corresponds to shaping an array pattern with phase only and may represent a severe problem if sharp pattern skirts are required. It can be avoided with extended feeds.

Multiple Beams and Extended Feeds.^{19–21} A feed at the focal point of a parabola forms a beam parallel to the focal axis. Additional feeds displaced from the focal point form additional beams at angles from the axis. This is a powerful capability of the reflector antenna to provide extended coverage with a modest increase in hardware. Each additional beam can have nearly full gain, and adjacent beams can be compared with each other to interpolate angle.

A parabola reflects a spherical wave into a plane wave only when the source is at the focus. With the source off the focus, a phase distortion results that in-



FIG. 6.14 ASR-9 shaped reflector with offset feed and an air traffic control radar beacon system (ATCRBS) array mounted on top. (*Courtesy Westinghouse Electric Corporation.*)



FIG. 6.15 Patterns for off-axis feeds.

creases with the angular displacement in beamwidths and decreases with an increase in the focal length. Figure 6.15 shows the effect of this distortion on the pattern of a typical dish as a feed is moved off axis. A flat dish with a long focal length minimizes the distortions. Progressively illuminating a smaller fraction of the reflector as the feed is displaced accomplishes the same purpose.

Two secondary effects are influential in the design of extended feeds. If an off-axis feed is moved parallel to the focal axis, the region of minimum distortion moves laterally in the reflector. At the same time, if the reflector is a paraboloid of revolution, the focus in the orthogonal plane (usually the azimuth) is altered. For the reflector region directly in front of the displaced feed, it has been found that both planes are improved by moving progressively back from the focal plane. This is clearly illustrated in the side view of the AN/TPS-43 antenna of Fig. 6.16. If that feed is examined carefully, one can also see that off-axis feeds become progressively larger so as to form progressively wider elevation beams that maintain a nearly constant number of beamwidths off axis. This is often made possible by radar coverage requirements having reduced range at wide elevation angles.



FIG. 6.16 AN/TPS-43 multiple-beam antenna. (Courtesy Westinghouse Electric Corporation.)

For some purposes the extended feed is not placed about the focal plane at all. If we consider the reflector as a collector of parallel rays from a range of angles and examine the converging ray paths (Fig. 6.17) it is evident that a region can be found that intercepts most of the energy. A feed array in that region driven with suitable phase and amplitude can therefore efficiently form beams at any of the angles. This ability has been used in various systems as a means of forming agile beams over a limited sector. It has also been used as a means of shaping beams and of forming very low sidelobe illumination functions. One such antenna is illustrated in Fig. 6.18.

Monopulse Feeds.^{22–25} Monopulse is the most common form of multiplebeam antenna, normally used in tracking systems in which a movable antenna keeps the target near the null and measures the mechanical angle, as opposed to a surveillance system having overlapping beams with angles measured from RF difference data.

Two basic monopulse systems, phase comparison and amplitude comparison, are illustrated in Fig. 6.19. The amplitude system is far more prevalent in radar antennas, using the sum of the two feed outputs to form a high-gain, low-sidelobe beam, and the difference to form a precise, deep null at boresight. The sum beam is used on transmit and on receive to detect the target. The difference port provides angle determination. Usually both azimuth and elevation differences are provided.

If a reflector is illuminated with a group of four feed elements, a conflict arises



FIG. 6.17 Extended feeds off the focal plane. (a) Geometry. (b) Feed detail.



FIG. 6.18 Low-sidelobe reflector using an extended feed. (Courtesy Westinghouse Electric Corporation.)



FIG. 6.19 Monopulse antennas. (a) Phase. (b) Amplitude.



FIG. 6.20 Conflicting taper requirements for sum and difference horn designs (*H* plane illustrated).

between the goals of high sum-beam efficiency and high difference-beam slopes. The former requires a small overall horn size, while the latter requires large individual horns (Fig. 6.20). Numerous methods have been devised to overcome this problem, as well as the associated high-difference sidelobes. In each case the comparator is arranged to use a different set of elements for the sum and difference beams. In some cases this is accomplished with oversized feeds that permit two modes with the sum excitation. Hannan²⁴ has tabulated results for several configurations, as summarized in Table 6.1.

Multiple-Reflector Antennas.^{26–31} Some of the shortcomings of paraboloidal reflectors can be overcome by adding a secondary reflector. The contour of the added reflector determines how the power will be distributed across the primary reflector and thereby gives control over amplitude in addition to phase in the aperture. This can be used to produce very low spillover or to produce a specific low-sidelobe distribution. The secondary reflector may also be used to

Type of horn	H plane		E plane	Sidelobes, dB		Feed
	Efficiency	Slope	Slope	Sum	Difference	shape
Simple four- horn	0.58	1.2	1.2	19	10	Ħ
Two-horn dual-mode	0.75	1.6	1.2	19	10	\square
Two-horn triple-mode	0.75	1.6	1.2	19	10	
Twelve-horn	0.56	1.7	1.6	19	19	HHH
Four-horn triple-mode	0.75	1.6	1.6	19	19	

TABLE 6.1 Monopulse Feedhorn Performance

relocate the feed close to the source or receiver. By suitable choice of shape, the apparent focal length can be enlarged so that the feed size is convenient, as is sometimes necessary for monopulse operation.

The Cassegrain antenna (Fig. 6.21), derived from telescope designs, is the most common antenna using multiple reflectors. The feed illuminates the hyperboloidal subreflector, which in turn illuminates the paraboloidal main reflector. The feed is placed at one focus of the hyperboloid and the paraboloid focus at the other. A similar antenna is the gregorian, which uses an ellipsoidal subreflector in place of the hyperboloid.

The parameters of the Cassegrain antenna are related by the following expressions:

$$\tan \psi_r / 2 = 0.25 D_m / f_m \tag{6.17}$$

$$1/\tan \psi_v + 1/\tan \psi_r = 2f_s/D_s$$
 (6.18)

$$1 - 1/e = 2 L_r / f_c \tag{6.19}$$

where the eccentricity e of the hyperboloid is given by

$$e = \sin \left[(\psi_v + \psi_r)/2 \right] / \sin \left[(\psi_v - \psi_r)/2 \right]$$
(6.20)

The equivalent-paraboloid^{4,26} concept is a convenient method of analyzing the radiation characteristics in which the same feed is assumed to illuminate a virtual reflector of equal diameter set behind the subreflector. The equation

$$f_e = D_m / (4 \tan \psi_r / 2)$$
 (6.21)

defines the equivalent focal length, and the magnification m is given by

$$m = f_e / f_m = (e + 1) / (e - 1)$$
(6.22)

Thus the feed may be designed to produce suitable illumination within subtended angles $\pm \psi_r$ for the longer focal length. Typical aperture efficiency can be better than 50 to 60 percent.

Aperture blocking can be large for symmetrical Cassegrain antennas. It may



FIG. 6.21 The Cassegrain reflector antenna. (a) Schematic diagram. (b) Geometry. (c) Offset dual reflector.

be minimized by choosing the diameter of the subreflector equal to that of the feed. $^{\rm 26}$ This occurs when

$$D'_{S} \simeq \sqrt{2f_{m}\lambda/k} \tag{6.23}$$

where k is the ratio of the feed-aperture diameter to its effective blocking diameter. Ordinarily k is slightly less than 1. If the system allows, blocking can be reduced significantly by using a polarization-twist reflector and a subreflector made of parallel wires.^{3,26} The subreflector is transparent to the secondary beam with its twisted polarization.

In the general dual-reflector case, blockage can be eliminated by offsetting both the feed and the subreflector (Fig. 3.21c). With blockage and supporting struts and spillover virtually eliminated, this is a candidate for very low sidelobes.³² It can be used in conjunction with an extended feed to provide multiple or steerable beams.³³

Special-Purpose Reflectors. Several types of antennas are occasionally used for special purposes. One such antenna is the spherical reflector,³⁴ which can be scanned over very wide angles with a small but fixed phase error known as spherical aberration. The basis of this antenna is that, over small regions, a spherical surface viewed from a point halfway between the center of the circle and the surface is nearly parabolic. If the feed is moved circumferentially at constant radius R/2, where R = the radius of the circular reflector surface, the secondary beam can be steered over whatever angular extent the reflector size permits. In fact, 360° of azimuth steering may be accomplished if the feed polarization is tilted 45° and the reflector is formed of conducting strips parallel to the polarization. The reflected wave is polarized at right angles to the strips on the opposite side. This antenna is known as a Helisphere.³⁵

If the scanning is in azimuth only, the height dimension of the reflector may be parabolic for perfect elevation focus. This is the parabolic torus,^{3,4} which has been used in fixed radar installations.

6.4 FEEDS

Because most radar systems operate at microwave frequencies (L band and higher), feeds for reflector antennas are typically some form of flared waveguide horn. At lower frequencies (L band and lower) dipole feeds are sometimes used, particularly in the form of a linear array of dipoles to feed a parabolic-cylinder reflector. Other feed types used in some cases include waveguide slots, troughs, and open-ended waveguides, but the flared waveguide horns are most widely used.

Paraboloidal reflectors (in the receive mode) convert incoming plane waves into spherical phase fronts centered at the focus. For this reason, feeds must be point-source radiators; i.e., they must radiate spherical phase fronts (in the transmit mode) if the desired directive antenna pattern is to be achieved. Other characteristics that a feed must provide include the proper illumination of the reflector with a prescribed amplitude distribution and minimum spillover and correct polarization with minimum cross polarization; the feed must also be capable of handling the required peak and average power levels without breakdown under all operational environments. These are the basic factors involved in the choice or design of a feed for a reflector antenna. Other considerations include operating bandwidth and whether the antenna is a single-beam, multibeam, or monopulse antenna.

Rectangular (pyramidal) waveguide horns propagating the dominant TE_{01} mode are widely used because they meet the high power and other requirements, although in some cases circular waveguide feeds with conical flares propagating the TE_{11} mode have been used. These single-mode, simply flared horns suffice for pencil-beam antennas with just one linear polarization.

When more demanding antenna performance is required, such as polarization diversity, multiple beams, high beam efficiency, or ultralow sidelobes, the feeds become correspondingly more complex. For such antennas segmented, finned, multimode, and/or corrugated horns are used. Figure 6.22 illustrates a number of feed types, many of which are described in more detail in antenna references.^{3,36,37}



FIG. 6.22 Various types of feeds for reflector antennas.

6.5 REFLECTOR ANTENNA ANALYSIS

In calculating the antenna radiation pattern, it is assumed that the reflector is a distance from the feed such that the incident field on it has a spherical wavefront. There are two methods^{2.38} for obtaining the radiation field produced by a reflector antenna. The first method, known as the current-distribution method, calculates the field from the currents induced on the reflector because of the primary

RADAR HANDBOOK

field of the feed. The second method, known as the aperture-field method, obtains the far field from the field distribution in the aperture plane. Both the current and aperture-field distributions are obtained from geometrical optics considerations. The two methods predict the same results in the limit $\lambda/D \rightarrow ?$. However, in contrast to the aperture-field method, the current-distribution method can explain the effect of antenna surface curvature on the sidelobe levels and on the polarization. While the aperture-field method is handy for approximations and estimates, another problem is that it assumes that the reflection from the surface forms a planar wavefront. This is true for a paraboloidal reflector fed at its focal point, but otherwise it is not true. For this reason, the analysis that follows is devoted to the more general current-distribution method, or inducedcurrent approach.

Although most antennas are reciprocal devices (have the same patterns in receive and transmit), analysis typically follows the transmit situation in which the signal begins at the feed element and its progress is tracked to the far field. Also at the feed, the polarization is in its purest form, so the vector properties are best known at this point and are described in many textbooks. In the analyses that follow, the constants are usually stripped away from the textbook versions since the antenna designer's primary goal in analysis is normally to determine the antenna's gain and pattern for main and cross polarizations. Therefore, the designer will normally integrate the power radiated from a feed into a sphere to determine the normalization factors needed. The magnetic field \overline{H} of the feed is chosen because it leads to the reflector surface current \overline{J} via the normal to the surface \hat{n} , by $\overline{J} = \hat{n} \times \overline{H}$.



FIG. 6.23 Feed and normalization geometry.

The primary feed is assumed to radiate with the *H* field, $\overline{H}(\hat{v})$, perpendicular to the direction unit vector \hat{v} (Fig. 6.23). It is dependent on feed type, say, horn or dipole. This *H* field is normalized such that the total power into a surrounding sphere (the magnitude of the tangent field squared) is equal to 1 W. This may be done by numerical integration using as much symmetry as possible to reduce computation time.

Radar reflectors are normally used to shape and distribute energy, which is more complicated than the case of the symmetrical paraboloidal reflector. Thus, where the focused paraboloid reflects into a common plane over the entire reflector, the shaped reflector focuses into many planes and the most general analysis is to treat the problem

as an incremental summation of E fields. Another advantage of this analysis method is that the reflector outline can also be most general.

The surface of the reflector is divided into rectangular grid regions of area dA (Fig. 6.24), which intercept the feed-radiated field. The surface current then is the cross product of the H field with the normal to the surface \hat{n} modified by differential area and a phase term,

$$\overline{J} = \hat{n} \times \overline{H}(\hat{v}) \, (\hat{v} \cdot \hat{n}) \, e^{-ikr} \, dA/4\pi r \tag{6.24}$$



FIG. 6.24 Reflector geometry.

where r = the distance from the feed to the reflecting surface and $k = 2\pi/\lambda =$ the wavenumber.

Each reflector grid region represents the reflection of a small uniformly illuminated section. It has a gain factor and also a direction of reflection, which follows Snell's law. The direction of reflection \hat{s} can be written

$$\hat{s} = \hat{v} - 2 (\hat{n} \cdot \hat{v}) \hat{n}$$
 (6.25)

and the differential surface reflection at each grid region is modified by a pattern factor represented by a uniformly illuminated reflection steered in the direction of unit vector \hat{s} and determined in the pattern direction unit vector \hat{p} by

Pattern factor =
$$\frac{4\pi dA}{\lambda^2 |\hat{n} \cdot \hat{s}|} \frac{\sin \pi \Delta x (s_x - p_x)}{\pi \Delta x (s_x - p_x)} \frac{\sin \pi \Delta y (s_y - p_y)}{\pi \Delta y (s_y - p_y)}$$
(6.26)

This factor modifies the surface current \overline{J} as seen in the far field, projected in the direction of interest. At a distant spherical surface, vector \hat{p} is normal to the surface. Two vectors are determined at the surface for the polarizations of interest, both perpendicular to p and perpendicular to each other, considered main and cross-polarized directions. The dot product of each of these unit vectors with \overline{J} gives the field in the main and cross-polarized directions.

This pattern solution is found by a compromise between the number of grid regions and the time to compute all the parameters for each grid point. When the pattern is desired far off the pattern peak, one must consider the artificial grating lobes created by the computation method itself, in which case the grid density must be increased. With grid size Δx , the artificially induced grating lobe will appear at the angle found from $\sin \theta = \lambda / \Delta x$. Frequently, the user of such computational tools will trade off grid density in the orthogonal plane to enhance computation accuracy in the plane of interest.

Typically, the computer time-consuming operations in this type of pattern computation are trigonometric functions, sines and cosines, and square roots used in length and thus phase calculations. Extensive techniques are usually devised to minimize repetitive calculations through the use of arrays containing the unit vectors in the pattern directions of interest, polarization vectors for those directions, and symmetry.

The literature contains many articles showing how geometric theory of diffraction (GTD) techniques can be used to compute reflector patterns.^{7,39} The problem one encounters with GTD is in generalizing the situation, such as an irregular antenna outline. The primary use one finds for GTD is in defining the antenna's operation in the back hemisphere, but for many antennas the irregularity of the edges requires an agonizingly complex description of the antenna. This is often found to be impractical to implement into a GTD analysis. Sometimes simple analyses are performed at special angles of interest.

6.6 SHAPED-BEAM ANTENNAS

Rotating search radars typically require antenna patterns which have a narrow azimuth beamwidth for angular resolution and a shaped elevation pattern designed to meet multiple requirements. When circular polarization is also one of the system requirements, a shaped reflector is almost always the practical choice, since circularly polarized arrays are quite expensive.

A typical range coverage requirement might look like that shown in Fig. 6.25. At low elevation angles, the maximum range is the critical requirement. Above the height-range limit intersection, altitude becomes the governing requirement, resulting in a cosecant-squared pattern shape. At still higher elevation angles,



FIG. 6.25 Typical two-way coverage requirement example.